THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE V TECHNOLOGIES IN MACHINE BUILDING, ISSN 1221-4566
2002

CONVOLUTE "FREDASCON" HOB TOOLS PROFILING BY HELICAL CONTINUE SHARPENING

Nicu Căpățână, Emil Țâru
Universitatea "Dunărea de Jos" din Galați

ABSTRACT

This paper presents the profiling method of the cutting edges of a "Fredascon" hob-tool of ZN1 and ZN2 convolute type (STAS 6845-82), by helical grinding of the teeth flanks, directly on the tool body.

The profiling method by continue helical sharpening of the hob tools of this kind eliminates the classical relief grid operation and assures other important advantages like: the increase of profile precision of the finishing tool, the increase of the workpiece surface quality, the increase of the tool edges durability and the sharpening number [1],[2].

1. Introduction

The versions of hob tools with moving teeth of "Fredascon" type offer the possibility of edges profiling by continue helical sharpening of top and side flanks, through one positioning on the tool body due to the teeth eccentricity related to the axial plane of the milling cutter.

In the profiling position, by continue sharpening, the hob tool teeth are brought with the flanks on the helical surfaces, eliminating the relief, special machine tools, which are, generally, bought from abroad.

After the helical grinding of the flanks of the hob tool teeth, with profiled grinding disc-tools, by rotating the teeth 180° around the axis we are obtaining the necessary cutting angles and the corresponding profile of the active edges [1].

The paper presents the profiling method by helical continue sharpening of "Fredascon" hob tool of ZN1 and ZN2 convolute type, for the cylindrical gear with evolvente profile.

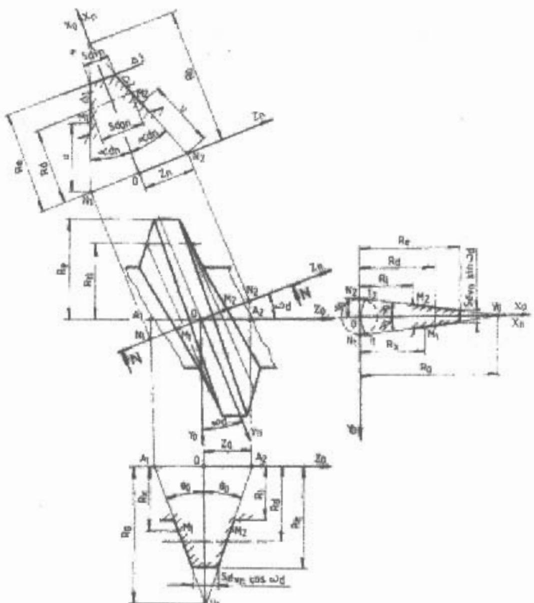

Fig. 1 Continue generation of helical convolute surfaces

2. The hob tool surface

In the mobile system of reference $X_nY_nZ_n$, with the origin in the point O (fig.1), having the X_nZ_n plane normal to the division helice of the tool tooth, the parametric equations of the Δ_1 and Δ_2 lines that are generating the helical flanks of the wrapping hob of ZN1 convolute type, are:

$$\Delta_{1,2}\left|\begin{aligned}&X_n = u\cos\alpha_{dn};\\&Y_n = 0;\\&Z_n = \mp\left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right).\end{aligned}\right. \quad (1)$$

For the ZN2 convolute hob, with a linear profile in the section normal to the tooth, the equation (1) becomes:

$$\Delta'_{1,2}\left|\begin{array}{l} X_n = u\cos\alpha_{gn};\\ Y_n = 0;\\ Z_n = \mp\left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right). \end{array}\right. \qquad (1')$$

The transformation: $X = \alpha^T(\omega_d)\cdot X_n$, (2)

where $\alpha^T(\omega_d) = \begin{Vmatrix} 1 & 0 & 0\\ 0 & \cos\omega_d & -\sin\omega_d\\ 0 & \sin\omega_d & \cos\omega_d \end{Vmatrix}$, the equations (1) and (1') are reported to the mobile system $X_0Y_0Z_0$ that is related to the hob assembles for the teeth generation. After replacement, we are obtaining:

$$\Delta_{01,2}\left|\begin{array}{l} X_0 = u\cos\alpha_{dn};\\ Y_0 = \pm\left(\dfrac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right)\sin\omega_d;\\ Z_0 = \mp\left(\dfrac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right)\cos\omega_d. \end{array}\right. \qquad (3)$$

$$\Delta'_{01,2}\left|\begin{array}{l} X_0 = u\cos\alpha_{gn};\\ Y_0 = \pm\left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right)\sin\omega_d;\\ Z_0 = \mp\left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right)\cos\omega_d. \end{array}\right. \qquad (3')$$

The convolute helical surfaces of the cutting edges of the hob teeth are generated at tooth generation by the Δ_1 and Δ_2 lines in the helical movement of V_0 axis and p the helical parameter, movement described by the equation (4):

$$x_0 = \omega_3^T(\varphi)X_0 + p\varphi\bar{k} \qquad (4)$$

which u and φ are variable parameters of the surface ,

$$p = \frac{p_a}{2\pi} = \frac{m_n}{2\cos\omega_d}\left(\omega_d = \arcsin\frac{m_n}{R_d}\right),$$

after development and replacement, we are obtaining:

$$\Sigma_{0_{1,2}}\begin{cases} x_0 = u\cos\alpha_{dn}\cos\varphi \mp \left(\dfrac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right)\sin\omega_d\sin\varphi;\\ y_0 = u\cos\alpha_{dn}\sin\varphi \pm \left(\dfrac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right)\sin\omega_d\cos\varphi;\\ z_0 = \mp\left(\dfrac{S_n}{2} + R_d tg\alpha_{dn} - u\sin\alpha_{dn}\right)\cos\omega_d + p\varphi. \end{cases} \qquad (5)$$

$$\Sigma'_{0_{1,2}}\begin{cases} x_0 = u\cos\alpha_{gn}\cos\varphi \mp \left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right)\sin\omega_d\sin\varphi;\\ y_0 = u\cos\alpha_{gn}\sin\varphi \pm \left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right)\sin\omega_d\cos\varphi;\\ z_0 = \mp\left(\dfrac{S_n}{2} + R_d tg\alpha_{gn} - u\sin\alpha_{gn}\right)\cos\omega_d + p\varphi. \end{cases} \qquad (5')$$

3. Releasing Surface of the Hob

For the hob with a positiv γ_v angle, the equations of the face tooth surfaces situated on the helical channels for chips, channels that are normal to the chip helices of the hob tool, are:

$$\Sigma_c \begin{cases} x_c = \sqrt{R_e^2 \sin^2 \gamma_v + u_c^2 \sin^2 \beta_c} \cdot \cos\varphi_c; \\ y_c = \sqrt{R_e^2 \sin^2 \gamma_v + u_c^2 \sin^2 \beta_c} \cdot \sin\varphi_c; \\ z_c = p_c(\theta_c - \varphi_c) - u_c \cos\beta_c. \end{cases} \quad (6)$$

where $\beta_c = arctg\left(\frac{R_e \sin\gamma_v}{p_c}\right)$ and

$$\theta_c = arctg\left(\frac{u_c \sin\beta_c}{R_e \sin\gamma_v}\right) \text{ with } p_c = \frac{p_k}{2\pi} = \frac{R_d}{tg\omega_d}.$$

4. Cutting edges of the hob teeth

The intersection of the hob surface (convolute helical surface $\Sigma_{0_{1,2}}$) with the helical surface of the channels for the chips Σ_c, determine the side cutting edges of the teeth.

The intersection conditions:

$$u_c = \frac{\sqrt{u^2 \cos^2\alpha_{dn} + \left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)^2 \sin^2\omega_d - R_e^2 \sin^2\gamma_v}}{\sin\beta_c} \quad (7)$$

$$u'_c = \frac{\sqrt{u^2 \cos^2\alpha_{gn} + \left(\frac{S_n}{2} + R_d tg\alpha_{gn} - u \sin\alpha_{gn}\right)^2 \sin^2\omega_d - R_e^2 \sin^2\gamma_v}}{\sin\beta_c} \quad (7')$$

$$\varphi_c = \frac{\pm\left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)\cos\omega_d + p_c\theta_c - u_c \cos\beta_c}{p + p_c}, \quad (8)$$

$$\varphi'_c = \frac{\pm\left(\frac{S_n}{2} + R_d tg\alpha_{gn} - u \sin\alpha_{gn}\right)\cos\omega_d + p_c\theta_c - u_c \cos\beta_c}{p + p_c}, \quad (8')$$

attached to the equations (5), determines the parametric equations of the curved cutting edges $\Gamma_{0_{1,2}}$.

$$\Gamma_{0_{1,2}} \begin{cases} \begin{cases} X_0 = \sqrt{u^2 \cos^2\alpha_{dn} + \left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)^2 \sin^2\omega_d} \cdot \cos\varphi_c \\ Y_0 = \sqrt{u^2 \cos^2\alpha_{dn} + \left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)^2 \sin^2\omega_d} \cdot \sin\varphi_c \\ Z_0 = \frac{pp_c\theta_c \mp p_c\left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)\cos\omega_d - p_c u_c \cos\beta_c}{p + p_c} \end{cases} \\ u_c = \frac{\sqrt{u^2 \cos^2\alpha_{dn} + \left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)^2 \sin^2\omega_d - R_e^2 \sin^2\gamma_v}}{p + p_c} \\ \varphi_c = \frac{\pm\left(\frac{S_n}{2} + R_d tg\alpha_{dn} - u \sin\alpha_{dn}\right)\cos\omega_d + p_c\theta_c - u_c \cos\beta_c}{p + p_c} \end{cases} \quad (9)$$

$$\Gamma'_{0_{1,2}}\begin{cases}\begin{cases}X_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\cos\varphi_c\\ Y_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\sin\varphi_c\\ Z_0=\dfrac{pp_c\theta_c\mp p_c\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d-p_c u_c\cos\beta_c}{p+p_c}\end{cases}\\ u_c=\dfrac{\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d-R_e^2\sin^2\gamma_v}}{p+p_c}\\ \varphi_c=\dfrac{\pm\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d+p_c\theta_c-u_c\cos\beta_c}{p+p_c}\end{cases}\tag{9'}$$

5. Helical surface generated by continue sharpening of the flanks

By the coordinate transformation:

$$X=\omega_3^T(\gamma)X_0-A\tag{10}$$

where $\omega_3^T(\gamma)=\begin{Vmatrix}\cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{Vmatrix}$, $A=\begin{Vmatrix}0\\ 2a\\ 0\end{Vmatrix}$

and $\gamma=arcsin[(a+h)/R_e]$, we are reporting the parametric equations of the cutting edges to the mobile system of references XYZ, attached to the "Fredascon" hob having the teeth situated in the position necessary for the continue helical grinding of the flanks.

After calculations and replacement, we are obtaining:

$$\Gamma_{1,2}\begin{cases}\begin{cases}X_0=\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d}\cdot\cos(\varphi_c+\gamma);\\ Y_0=\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d}\cdot\sin(\varphi_c+\gamma)-2a;\\ Z_0=\dfrac{pp_c\theta_c\mp p_c\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)\cos\omega_d-p_c u_c\cos\beta_c}{p+p_c}\end{cases}\\ u_c=\dfrac{\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d-R_e^2\sin^2\gamma_v}}{p+p_c}\\ \varphi_c=\dfrac{\pm\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)\cos\omega_d+p_c\theta_c-u_c\cos\beta_c}{p+p_c}\end{cases}\tag{11}$$

$$\Gamma'_{1,2}\begin{cases}\begin{cases}X_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\cos(\varphi_c+\gamma)\\ Y_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\sin(\varphi_c+\gamma)-2a;\\ Z_0=\dfrac{pp_c\theta_c\mp p_c\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d-p_c u_c\cos\beta_c}{p+p_c}\end{cases}\\ u_c=\dfrac{\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d-R_e^2\sin^2\gamma_v}}{p+p_c}\\ \varphi_c=\dfrac{\pm\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d+p_c\theta_c-u_c\cos\beta_c}{p+p_c}\end{cases}\qquad(11')$$

In the helical movement of $\overline{V}$ axis and helical parameter p, described by the transformation:

$$x=\omega_3^T(\varphi)X+p\varphi\bar{k}\qquad(12)$$

the $\Gamma_{1,2}$ curves will generate the continue sharpening of the flanks of the "Fredascon" hob on helical surfaces.

By replacements, we are obtaining:

$$\Sigma_{A_{1,2}}\begin{cases}\begin{cases}X_0=\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d}\cdot\cos(\varphi_c+\gamma+\varphi)+2a\sin\varphi;\\ Y_0=\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d}\cdot\sin(\varphi_c+\gamma+\varphi)-2a\cos\varphi;\\ Z_0=\dfrac{pp_c\theta_c\mp p_c\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)\cos\omega_d-p_c u_c\cos\beta_c}{p+p_c}+p\varphi\end{cases}\\ u_c=\dfrac{\sqrt{u^2\cos^2\alpha_{dn}+\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)^2\sin^2\omega_d-R_e^2\sin^2\gamma_v}}{p+p_c}\\ \varphi_c=\dfrac{\pm\left(\frac{S_n}{2}+R_d tg\alpha_{dn}-u\sin\alpha_{dn}\right)\cos\omega_d+p_c\theta_c-u_c\cos\beta_c}{p+p_c}\end{cases}\qquad(13)$$

$$\Sigma'_{A_{1,2}}\begin{cases} X_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\cos(\varphi_c+\gamma+\varphi)+2a\sin\varphi \\ Y_0=\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d}\cdot\sin(\varphi_c+\gamma+\varphi)-2a\cos\varphi; \\ Z_0=\dfrac{pp_c\theta_c\mp p_c\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d-p_c u_c\cos\beta_c}{p+p_c}+p\varphi \\ u_c=\dfrac{\sqrt{u^2\cos^2\alpha_{gn}+\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)^2\sin^2\omega_d-R_e^2\sin^2\gamma_v}}{p+p_c} \\ \varphi_c=\dfrac{\pm\left(\frac{S_n}{2}+R_d tg\alpha_{gn}-u\sin\alpha_{gn}\right)\cos\omega_d+p_c\theta_c-u_c\cos\beta_c}{p+p_c} \end{cases} \quad (13')$$

The obtained equations can be used for hob with zero rake angle or with plane teeth faces.

6. Disc tool profiling for helical continue sharpening of the hob

The determination of the S revolution surface of the abrasive tool from the wrapping up condition with the $\Sigma_{A_{1,2}}$ surface is supposing that, S is accepting a characteristic curve common to the grinding helical surface of the flanks of the convolute "Fradascon" hob.

Condition for the determination of the characteristic curve in the usual case, the axis of the S disc-tool surface is normal on the helical line of the exterior diameter of the Σ_A flanks.

The condition for the determination of the C characteristic is given by the condition that the $\overline{N}_{\Sigma_A}$, $\overline{A}$ and $\bar{r}_1$ vectors belong to the same plane:

$$\begin{vmatrix} N_{xA} & N_{yA} & N_{zA} \\ 0 & -\sin\omega_e & \cos\omega_e \\ x_A-b & y_A & z_A \end{vmatrix}=0 \quad (14)$$

which is implying that:

$$(x_A-b)(N_{yA}\cos\omega_e+N_{zA}\sin\omega_e)-N_{xA}(y_A\cos\omega_e+z_A\sin\omega_e)=0 \quad (15)$$

where $\omega_e=arctg\frac{p_a}{2\pi R_e}=arctg\frac{p}{R_e}$.

The equation (15) associated to the parametric equations of the side flanks helical surfaces $\Sigma_{A_{1,2}}$ (13) determines the characteristic curve on the S disc-tool surface.

After replacements, for the two side flanks helical surfaces grinded by continue helical surfaces with profiled grinding disc-tool, the condition for characteristic tool determination (15) establishes a relation between the u and φ variable parameters as follows:

$$A(\varphi)u^2+B(\varphi)u+C(\varphi)=0 \quad (16)$$

which can be written as:

$$u=u(\varphi) \quad (17)$$

7. The axial profile of the disc-tool

The characteristic curve can be reported to the $x_1y_1z_1$ system of reference, with S surface, by the change of coordinates:

$$\begin{Vmatrix} x_1 \\ y_1 \\ z_1 \end{Vmatrix}=\begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_e & \sin\omega_e \\ 0 & -\sin\omega_e & \cos\omega_e \end{Vmatrix}\cdot\begin{Vmatrix} x_A(\varphi)-b \\ y_A(\varphi) \\ z_A(\varphi) \end{Vmatrix} \quad (18)$$

The characteristic curve equation becomes:

$$C_1 \begin{cases} x_1 = x_A(\varphi) - b; \\ y_1 = y_A(\varphi)\cos\omega_e + z_A(\varphi)\sin\omega_e; \\ z_1 = -y_A(\varphi)\sin\omega_e + z_A(\varphi)\cos\omega_e. \end{cases} \quad (19)$$

By rotating the characteristic C_1 around the $\overline{A}$ axis we are generating the disc-tool surface, S. The axial profile of this surface, in a S system of coordinates, H is given by the generating curve:

$$G \begin{cases} R = \sqrt{x_1^2(\varphi) + y_1^2(\varphi)}; \\ H = z_1(\varphi) \end{cases} \quad (20)$$

where x_1, y_1, z_1 are given by the equation (19).
Knowing the axial section profile of the revolution surface is used for the control of profiling precision and the grinding disc-tool, S .
The variation field of the u parameter are:
$u_{min} = R_i / \cos\alpha_{dn}$ and $u_{max} = R_e / \cos\alpha_{dn}$.

8.Calculation examples

We are presenting an calculation example of the axial profile of the disc-tool for continue sharpening of the flanks of the "Fredascon" convolute hob with the following characteristics:
normal module: m_n=10mm;
presure angle: $\alpha_{dn} = 20^\circ$;
exterior diameter: D_e=226mm;
pitch diameter: D_d=197mm;
tooth eccentricity: a=6mm;
top heightens of tooth: h=10mm;
normal pitch: $p_n = \pi m_n = 10 \cdot \pi = 31{,}415926$ mm
the thickness of the hob tooth on the pitch diameter;
the angle of the helical line:

$$\omega_d = \arcsin m_n / D_{d_s} = 2^\circ 54' 39''$$

axial pitch of the hob: $p_a = \dfrac{\pi m_n}{\cos\omega_d} = 31{,}459mm$.

the pitch of the helical channels for the chips $p_k = \pi D_d ctg\omega_d = 12176mm$.

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