## PROFILING TOOL GENERATING METHOD, USING THE PROFILE MEASUREMENT OF ROTOR OF SCREW COMPRESSOR COMPONENTS

L. C. Popa, I. Popa, V. Teodor, N. Baroiu

Dunarea de Jos University of Galati, Department of Manufacturing, Robotics and Welding Engineering, Romania lacramioara.popa@ugal.ro

## ABSTRACT

Peripheral surface profiling of the disk and end mill cutters represents a problem of first degree enveloping.

In this paper, according to theorems referring to the Nicolaev theorem, we propose a profiling method of the primary peripheral surface of the disk and end mill cutter for the male rotor from the screw compressor component.

The constructive shape of the male rotor lobes, was achieved by measurement on machines Micro Hite 3D.

Also, based on a soft product, we present frontal applications of the axial shapes of the disk an end cutter for the male rotor, component of screw compressor, gear ratio 4/6.

**KEYWORDS:** screw compressor, polynomials Bézier, disk cutter, end mill cutter

## **1. Introduction**

The tool generating process of helical rotor lobes of the screw compressor, there may be need for tools for refurbishment operations of existing compressors.

In these circumstances, it is possible measurement of lobes rotor on the flanks on 3D machines, and the cloud of points obtained, leads to the algorithms for achieving profiling tools, bounded with peripheral revolution surfaces (disk cutter or end mill cutter), enveloping to the rotors surfaces.

Based on these dates, is also possible to build models of solid rotors in order to simulate the operation of screw compressors, or to improve their performance.

Figure 1 presents the male and female rotors, component of a screw compressor; ratio is i=4/6.

Measurement of helical rotor surfaces was made to the laboratories of the Department C.M.R.S. Mechanics of the University "Dunărea de Jos" of Galati.

Machine of measurement is type Micro Hite 3D with measurement accuracy of 0,001 mm.

The profiling problem of disk cutter, tools bounded with revolution surfaces, is according with fundamentals theorems of enveloping surfaces.



Fig. 1. Screw compressor rotors

The profiling of the disk cutter using the helicoidal surfaces envelope method is one well known. Problem is solved by first Olivier theorem and Gohman theorem [1], [2], [3].

We can also use one of complementary theorems: "The minim distance method" and "The family of substitutive circles" [3], or "The method of

plane trajectory generation" [6]. The graphics methods 2D or 3D can also solve the problem [4], [5].

## 2. Male rotor

Figure 2, shows the measurement process of male rotor on the machine Micro Hite 3D. This operation is realized in a plane section, parallel to the axis rotor. Axial plane of the rotor position is defined by measurement of cylindrical surfaces of revolution.

The coordinates measured are in the axial plane of the rotor.



Fig. 2. Male rotor on the machine Micro Hite 3D

Table 1, presents the measured coordinates of the male rotor flanks, AB and CD and also, on bottom profile of the teeth, arc BC (see also figure 3).

 Table 1. Measured coordinates of the male rotor
 profile

<u> </u>	/							
AB	$X_1[mm]$	$Y_1[mm]$	BC	$X_1[mm]$	$Y_1[mm]$	CD	$X_1[mm]$	$Y_1[mm]$
1	248.451	130.873	1	287.360	84.873	1	309.915	84.873
2	249.737	130.736	2	287.493	84.146	2	310.149	85.908
3	251.016	130.546	3	287.609	83.414	3	310.394	86.939
4	252.279	130.266	4	287.708	82.681	4	310.665	87.965
5	253.518	129.897	5	287.767	81.943	5	310.957	88.985
46	285.925	89.840	46	309.507	81.943	46	329.337	128.297
47	286.338	88.615	47	309.567	82.681	47	329.866	129.216
48	286.715	87.378	48	309.665	83.414	48	330.511	130.053
49	287.055	86.130	49	309.781	84.145	49	331.401	130.618
50	287 360	84 873	50	309 915	84 873	50	332.428	130 873

In Figure 3, is presented the axial profile of the rotor, as shown in the graphic interpretation of measured coordinates.



Determination of the step size was done by measuring the helical profile in two parallel planes, the same flank of profile leads to two profiles of the same flank of the rotor; in Table 2 are the coordinates of appropriate profile in the second plane of measurement.

**Table 2.** The measured coordinates of the profile in the supplementary plane

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Crt.	X[mm]	Y[mm]	Z[mm]				
no.							
1	80.060	293.140	100.000				
2	80.000	292.430	99.000				
3	80.000	291.720	98.000				
4	80.000	291.010	97.000				
5	80.000	291.010	97.000				
6	80.000	290.310	96.000				
7	79.990	289.600	95.000				
8	80.000	288.900	94.000				
9	80.000	288.210	93.000				
10	79.990	287.490	92.000				
11	79.990	286.790	91.000				

The two curves numerical represented in the two tables (Table 1 and Table 2), and substituted by Bézier polynomials, are intersected with a cylinder with the equation

$$x^2 + y^2 = r^2$$
. (1)

For r=125 mm, it determines two points located on the same cylinder with offset along its axis as follow

$$L = z_2 - z_1$$
. (2)

The coordinates  $z_1$  and  $z_2$  on the two curves, are determinate of polynomials Bézier, see also Table 1 and Table 2.

Helical parameter is determinate as follows, see Figure 4,

$$p = \frac{\left|z_2 - z_1\right|}{\varphi} \,. \tag{3}$$

The  $\varphi$  angle of the arc representing the projection of helix on the cylinder r, see Figure 4, is determined from the relationship

$$2 \cdot \sin \frac{\varphi}{2} = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} .$$
 (4)

For the measured rotor, results the step for the helix

$$P_{e_1} = 2\pi \cdot p_1 = 335.666 \text{ mm}.$$
 (5)



Fig. 4. The helical parameter determination

## 2.1. The disk cutter

We aim to determine the characteristic curve to the contact between the helical surface and the peripheral surface of disk tool, sides *AB* and *CD*, see also Figure 3 and Figure 5.



Fig. 5. The elementary helicoidal surface; the coordinates system

The matrix of coordinates of the axial profile of the rotor is is follow

$$G = \begin{vmatrix} X_{l_1} & Y_{l_1} \\ X_{l_2} & Y_{l_2} \\ \vdots & \vdots \\ X_{l_n} & Y_{l_n} \end{vmatrix}.$$
 (6)

The elementary segment (points from Table 1) is described by equations of the form:

$$M_{i}M_{i+1} \begin{vmatrix} X_{1} = X_{1i} + \lambda \cos\beta_{i}; \\ Y_{1} = Y_{1i} + \lambda \sin\beta_{i}, \end{vmatrix}$$
(7)

and

$$\tan \beta_{i} = \frac{\left|Y_{1i+1} - Y_{1i}\right|}{\left|X_{1i+1} - x_{1i}\right|}.$$
 (8)

Elementary helical surface, right screw, helical parameter  $p_1$  is described by  $M_iM_{i+1}$  segment and the equations as follow:

$$\begin{cases} X_{1} = X_{l_{i}} \cos \theta_{1} - Y_{l_{i}} \sin \theta_{1}; \\ Y_{1} = X_{l_{i}} \sin \theta_{1} + Y_{l_{i}} \cos \theta_{1}; \\ Z_{1} = p_{1}\theta_{1}; \end{cases}$$
(9)

 $\theta_1$  – angular variable parameter.

Directors of the tangent helical line (9) are as follow

$$\vec{\Gamma}_{M_{i}} = \frac{dX_{1}}{d\theta_{1}}\vec{i} + \frac{dY_{1}}{d\theta_{1}}\vec{j} + p_{1}\vec{k}$$
. (10)

We define the normal to the helical elementary surface, as follow

$$\vec{N}_{\Sigma} = \vec{T}_{M_i M_{i+1}} \times \vec{T}_{M_i}$$
(11)

or, (see equations (7), (9) and (10))

$$\vec{N}_{\Sigma} = N_{X_1}\vec{i} + N_{Y_1}\vec{j} + N_{Z_1}\vec{k}$$
 (12)

$$\begin{cases} N_{X_{1}} = -p_{1} \sin \beta_{i}; \\ N_{Y_{1}} = p_{1} \cos \beta_{i}; \\ N_{Z_{1}} = \sin \beta_{i} \left[ -X_{1_{i}} \sin \theta_{2} - Y_{1_{i}} \cos \theta_{1} \right] - \\ -\cos \beta_{i} \left[ X_{1_{i}} \cos \theta_{1} - Y_{1_{i}} \sin \theta_{1} \right]. \end{cases}$$

$$(13)$$

Nikolaev theorem can be writing, for elementary helical surface (9), as follow

$$\vec{N}_{\Sigma}, \vec{A}, \vec{r}_{l} = 0 \tag{14}$$

where:

where:

$$\vec{\mathbf{r}}_{l} = \begin{bmatrix} \mathbf{X}_{l_{i}} \cos \theta_{l} - \mathbf{Y}_{l_{i}} \sin \theta_{l} \end{bmatrix} \vec{\mathbf{i}} - \vec{\mathbf{a}} \cdot \vec{\mathbf{i}} +$$

$$(15)$$

$$\left[X_{l_i}\cos\theta_l - Y_{l_i}\sin\theta_l\right]\vec{j} - p_l\theta_l\vec{k},$$

$$\dot{\mathbf{A}} = -\sin\alpha \cdot \mathbf{j} + \cos\alpha \cdot \mathbf{k} \tag{16}$$

$$\vec{O}O_1 = a\vec{i}$$
 (17)

Points belonging to elementary helical surface assembly (pairs of values and  $\theta_I$ ,  $\lambda$ ) that satisfy the condition (14), see definitions (12), (15), (16) and (17), determines the characteristic curve shape,

$$X_{l}^{C} = \begin{cases} X_{l_{i}}^{C} \\ Y_{l_{i}}^{C} \\ Z_{l_{i}}^{C} \end{cases}, (i=1...m).$$
(18)

Reporting of these coordinates to the system of disk cutter,  $X_{IS}Y_{IS}Z_{IS}$ , the equations:

$$\begin{cases} H = Z_{1S}; \\ R = \sqrt{X_{1S}^2 + Y_{1S}^2}, \end{cases}$$
(19)

determine the axial section shape of disk cutter.

Similar proceeding is provided for profiles *AB* and *CD* (left and right sides), see Table 1 and Figures 6 and 7.



Fig. 6. Axial profile of the disk cutter, AB flank

If the distance between points on the flanks of the rotor is small enough (the amount segment  $M_iM_{i+1}$  is 0.1 mm of the order), we can approximate helical surfaces (9), with a helix ( $\lambda = 0$ )).

The shape and coordinates of axial section of the disk cutter, determined by the male rotor profile measured on side *AB*, are both, represented in Figure 6 and Table 3, for a the distance between the disk cutter axis and surface helical axis, a = 165 mm.

The result is a tilt angle of the outer helix of the male rotor,  $\beta = 23.155^{\circ}$ , and helical parameter  $p_1 = 53.460$  mm.

or AD flunk							
Crt.	R	Н	Crt.	R	Н		
no	[mm]	[mm]	no.	[mm]	[mm]		
1	35.165	-46.394	91	80.230	-8.185		
2	35.214	-45.708	92	80.871	-7.937		
3	35.271	-45.023	93	81.512	-7.689		
4	35.350	-44.340	94	82.154	-7.442		
5	35.457	-43.661	95	82.798	-7.202		
6	35.589	-42.987	99	83.442	-6.962		
7	35.761	-42.321	- 99	84.086	-6.722		
8	35.950	-41.660	99	84.732	-6.486		
9	36.173	-41.010	99	85.378	-6.252		
			100	86.024	-6.017		

**Table 3.** Coordinates of axial profile of the disk cutter for AB flank



Fig. 7. Axial profile of the disk cutter, CD flank

Figure 7 and Table 4, present the shape and the axial coordinates of the disk cutter axial section, for the measured profile of the rotor to *CD* flank.

**Table 4.** Coordinates of axial profile of the disk cutter for CD flank

Crt.	R	Н	Crt.	R	Н
no.	[mm]	[mm]	no.	[mm]	[mm]
1	86.445	5.865	91	38.183	27.700
2	85.890	6.063	92	37.669	27.988
3	85.334	6.259	93	37.155	28.277
4	84.779	6.455	94	36.641	28.565
5	84.223	6.653	95	36.132	28.861
6	83.668	6.850	96	35.659	29.213
7	83.113	7.048	97	35.244	29.621
8	82.559	7.249	98	34.927	30.118
9	82.005	7.451	99	34.733	30.667
			100	34.598	31.241

In Figure 8, is the solid model of the disk cutter, for *AB* and *CD* flanks.



Fig. 8. The model solid of the disk cutter for the male rotor

## 2. 2. End mill cutter

The axial profile foot area between two successive teeth of the rotor, flank *BC*, needs an end mill cutter because of its symmetry (see Figure 3 and Table 1).

End mill cutter is also a tool bordered by a revolution surface whose axis is normal and incident to the helical surface axis generated by the gap between the rotor lobes.

The principle of profiling method is similar to that described above with the specifications:

-the axis of end mill cutter, is superposed with axis  $X_{1S}$  ( $X_1$ ),

$$\mathbf{A} = \mathbf{i} ; \qquad (20)$$

- the vector  $\vec{r}_1$ , located in Nikolaev condition (14) takes the form

$$\vec{\mathbf{r}}_{l} = \begin{bmatrix} \mathbf{X}_{l_{i}} \cos \theta_{l} - \mathbf{Y}_{l_{i}} \sin \theta_{l} \end{bmatrix} \vec{\mathbf{i}} - a\vec{\mathbf{i}} + \begin{bmatrix} \mathbf{X}_{l_{i}} \cos \theta_{l} - \mathbf{Y}_{l_{i}} \sin \theta_{l} \end{bmatrix} \vec{\mathbf{j}} - p_{l}\theta_{l}\vec{\mathbf{k}},$$
(21)

where the  $X_{1i}$ ,  $Y_{1i}$  the are the coordinates measured corresponding Table 1.

The axial section of end mill cutter is:

$$\begin{cases} H = Z_{1S}; \\ R = \sqrt{X_{1S}^2 + Y_{1S}^2}; \end{cases}$$
(22)

where,

$$X_{1S} = X_{1i}; \quad Y_{1S} = Y_{1i}; \quad Z_{1S} = Z_{2i}.$$
 (23)

In Table 5 and Figure 8, has presented the coordinates of axial section of end mill cutter and its shape.

**Table 5.** Axial coordinates of the end mill cutter

 profile for male rotor
 Profile for male rotor

Crt.	R	Н	Crt.	R	Н
no.	[mm]	[mm]	no.	[mm]	[mm]
1	84.873	-11.277	91	81.607	10.870
2	84.512	-11.211	92	81.973	10.872
3	84.152	-11.145	93	82.338	10.902
4	83.790	-11.087	94	82.703	10.932
5	83.429	-11.030	95	83.066	10.981
6	83.066	-10.981	96	83.429	11.030
7	82.703	-10.932	97	83.790	11.087
8	82.338	-10.901	98	84.152	11.145
9	81.973	-10.872	99	84.512	11.211
10	81.607	-10.870	100	84.873	11.278



Fig. 8. Axial profile of the end mill cutter

In Figure 9, is the solid model of end mill cutter, determined by measurement of the male rotor profile, specific coordinates  $R_{I}$ ,  $H_{I}$ .



Fig. 9. The solid model of the end mill cutter

In Figure 10, is presented the solid model of the male rotor; the number of lobes is  $z_i = 4$ .



# Fig. 10. The solid model of the male rotor and of the gear

## 3. Product software for profiling primary peripheral surface by enveloping of disk cutter

The application (Java product) allows the configuration profile generated piece, the profile consists of several basic profiles (straight segment, circle arc). Tangent condition between profile components of the piece profiles apriori is solved by the user. It also allowed introducing discontinuities incremental profiles, of contact or tangent, to modelling on the discontinuities resulting end mill cutter profile.



Fig. 11. Applet - profiling of the primary surfaces of the disk cutter, for the male rotor

In the applet, it presents models of the primary peripheral surfaces of disk cutter to generate the male rotor (Figure 11), from the construction of the screw compressor. The program allows exporting profiles axial sections of the disk cutters coordinates in the file type *csv* (comma separated values), which can be further processed to achieve the solid model of tool.

## 4. Conclusions

Presented method allows to solve a problem referring to reverse engineering tools for profiling of the cutters bounded with surfaces revolution for a screw compressor rotor, known by measuring the actual coordinates of the axial profile.

The measured coordinates were presented; based on them and Nikolaev condition, we proposed an algorithm, for determining axial profile (template profile) of the disk cutter and end mill cutter tools for generating such a rotor.

We also proposed a solution for identifying the axial pitch (helical parameter) of the screw compressor rotor.

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### Metodă de profilare a sculelor generatoare utilizând forma discretă a unui profil măsurat al rotorului unui compresor elicoidal

#### -Rezumat-

Profilarea suprafeței periferice a sculelor de tip freză disc și cilindro-frontală reprezintă o problemă de înfășurare cu contact liniar.

În lucrare se prezintă o problemă de inginerie inversă referitoare la profilarea sculelor mărginite de suprafețe de revoluție (scula disc și scula cilindro-frontală) generatoare a rotorului conducător din componența unui compresor elicoidal 4/6.

Rotorul este cunoscut în baza măsurării acestuia pe o mașină de măsurat în coordonate de tip MicroHite 3D.

În acest fel, problema rezolvată se referă la o metodă de aproximare a unei suprafete măsurate prin polinoame Bezier.

S-au prezentat exemple numerice referitoare la determinarea secțiunii axiale a sculelor generatoare.